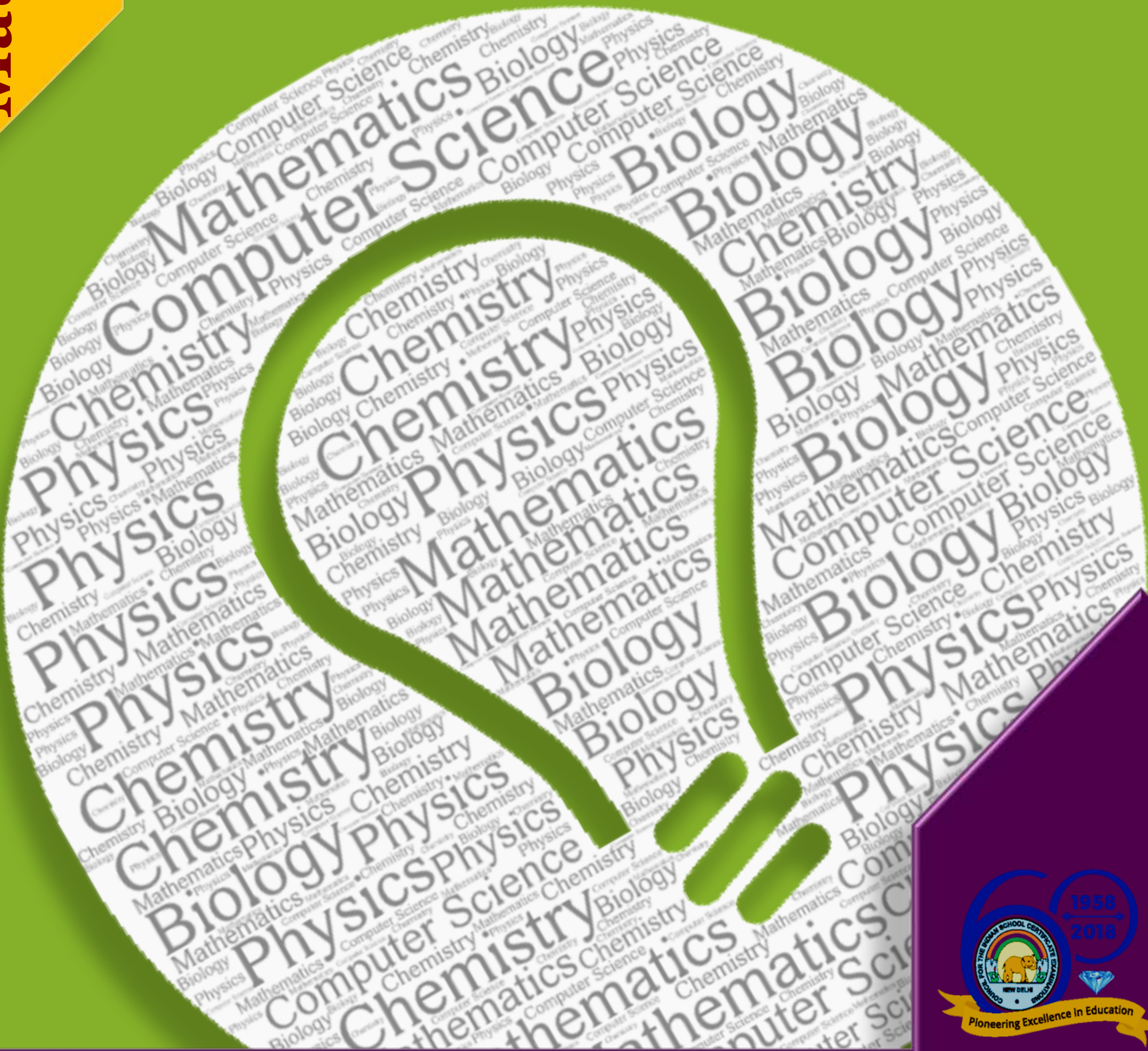
Analysis of Pupil Performance

ISC Year 2018 Examination

Sciences & Mathematics

MATHEMATICS



Research Development and Consultancy Division

Council for the Indian School Certificate Examinations
New Delhi

Year 2018

***Published by*:**

Research Development and Consultancy Division (RDCD)

Council for the Indian School Certificate Examinations

Pragati House, 3rd Floor

47-48, Nehru Place

New Delhi-110019

Tel: (011) 26413820/26411706
E-mail: council@cisce.org

FOREWORD

This document of the Analysis of Pupils' Performance at the ISC Year 12 and ICSE Year 10 Examination is one of its kind. It has grown and evolved over the years to provide feedback to schools in terms of the strengths and weaknesses of the candidates in handling the examinations.

We commend the work of Mrs. Shilpi Gupta (Deputy Head) of the Research Development and Consultancy Division (RDCD) of the Council and her team, who have painstakingly prepared this analysis. We are grateful to the examiners who have contributed through their comments on the performance of the candidates under examination as well as for their suggestions to teachers and students for the effective transaction of the syllabus.

We hope the schools will find this document useful. We invite comments from schools on its utility and quality.

Gerry Arathoon
Chief Executive & Secretary

October 2018

PREFACE

The Council has been involved in the preparation of the ICSE and ISC Analysis of Pupil Performance documents since the year 1994. Over these years, these documents have facilitated the teaching-learning process by providing subject/ paper wise feedback to teachers regarding performance of students at the ICSE and ISC Examinations. With the aim of ensuring wider accessibility to all stakeholders, from the year 2014, the ICSE and the ISC documents have been made available on the Council's website www.cisce.org.

The documents include a detailed qualitative analysis of the performance of students in different subjects which comprises of examiners' comments on common errors made by candidates, topics found difficult or confusing, marking scheme for each answer and suggestions for teachers/ candidates.

In addition to a detailed qualitative analysis, the Analysis of Pupil Performance documents for the Examination Year 2018 have a component of a detailed quantitative analysis. For each subject dealt with in the document, both at the ICSE and the ISC levels, a detailed statistical analysis has been done, which has been presented in a simple user-friendly manner.

It is hoped that this document will not only enable teachers to understand how their students have performed with respect to other students who appeared for the ICSE/ISC Year 2018 Examinations, but also provide information on how they have performed within the Region or State, their performance as compared to other Regions or States, etc. It will also help develop a better understanding of the assessment/ evaluation process. This will help teachers in guiding their students more effectively and comprehensively so that students prepare for the ICSE/ ISC Examinations, with a better understanding of what is required from them.

The Analysis of Pupil Performance document for ICSE for the Examination Year 2018 covers the following subjects: English (English Language, Literature in English), Hindi, History, Civics and Geography (History and Civics, Geography), Mathematics, Science (Physics, Chemistry, Biology), Commercial Studies, Economics, Computer Applications, Economic Applications, Commercial Applications.

Subjects covered in the ISC Analysis of Pupil Performance document for the Year 2018 include English (English Language and Literature in English), Hindi, Elective English, Physics (Theory), Chemistry (Theory), Biology (Theory), Mathematics, Computer Science, History, Political Science, Geography, Sociology, Psychology, Economics, Commerce, Accounts and Business Studies.

I would like to acknowledge the contribution of all the ICSE and the ISC examiners who have been an integral part of this exercise, whose valuable inputs have helped put this document together.

I would also like to thank the RDCD team of, Dr. M.K. Gandhi, Dr. Manika Sharma, Mrs. Roshni George and Mrs. Mansi Guleria who have done a commendable job in preparing this document.

Shilpi Gupta
Deputy Head - RDCD

October 2018

CONTENTS

INTRODUCTION

This document aims to provide a comprehensive picture of the performance of candidates in the subject. It comprises of two sections, which provide Quantitative and Qualitative analysis results in terms of performance of candidates in the subject for the ISC Year 2018 Examination. The details of the Quantitative and the Qualitative analysis are given below.

Quantitative Analysis

This section provides a detailed statistical analysis of the following:

- Overall Performance of candidates in the subject (Statistics at a Glance)
- State wise Performance of Candidates
- Gender wise comparison of Overall Performance
- Region wise comparison of Performance
- Comparison of Region wise performance on the basis of Gender
- Comparison of performance in different Mark Ranges and comparison on the basis of Gender for the top and bottom ranges
- Comparison of performance in different Grade categories and comparison on the basis of Gender for the top and bottom grades

The data has been presented in the form of means, frequencies and bar graphs.

Understanding the tables

Each of the comparison tables shows N (Number of candidates), Mean Marks obtained, Standard Errors and t-values with the level of significance. For t-test, mean values compared with their standard errors indicate whether an observed difference is likely to be a true difference or whether it has occurred by chance. The t-test has been applied using a confidence level of 95%, which means that if a difference is marked as 'statistically significant' (with * mark, refer to t-value column of the table), the probability of the difference occurring by chance is less than 5%. In other words, we are 95% confident that the difference between the two values is true.

t-test has been used to observe significant differences in the performance of boys and girls, gender wise differences within regions (North, East, South and West), gender wise differences within marks ranges (Top and bottom ranges) and gender wise differences within grades awarded (Grade 1 and Grade 9) at the ISC Year 2018 Examination.

The analysed data has been depicted in a simple and user-friendly manner.

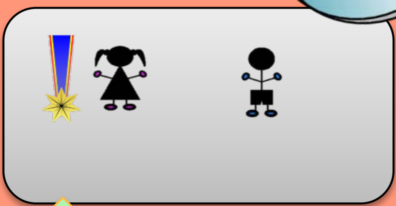
Given below is an example showing the comparison tables used in this section and the manner in which they should be interpreted.

Comparison on the basis of Gender

Gender	N	Mean	SE	t-value
Girls	2,538	66.1	0.29	11.91*
Boys	1,051	60.1	0.42	

*Significant at 0.05 level

Girls performed significantly better than boys.



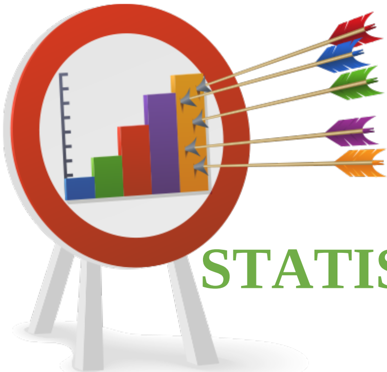
The table shows comparison between the performances of boys and girls in a particular subject. The t-value of 11.91 is significant at 0.05 level (mentioned below the table) with a mean of girls as 66.1 and that of boys as 60.1. It means that there is significant difference between the performance of boys and girls in the subject. The probability of this difference occurring by chance is less than 5%. The mean value of girls is higher than that of boys. It can be interpreted that girls are performing significantly better than boys.

The results have also been depicted pictographically. In this case, the girls performed significantly better than the boys. This is depicted by the girl with a medal.

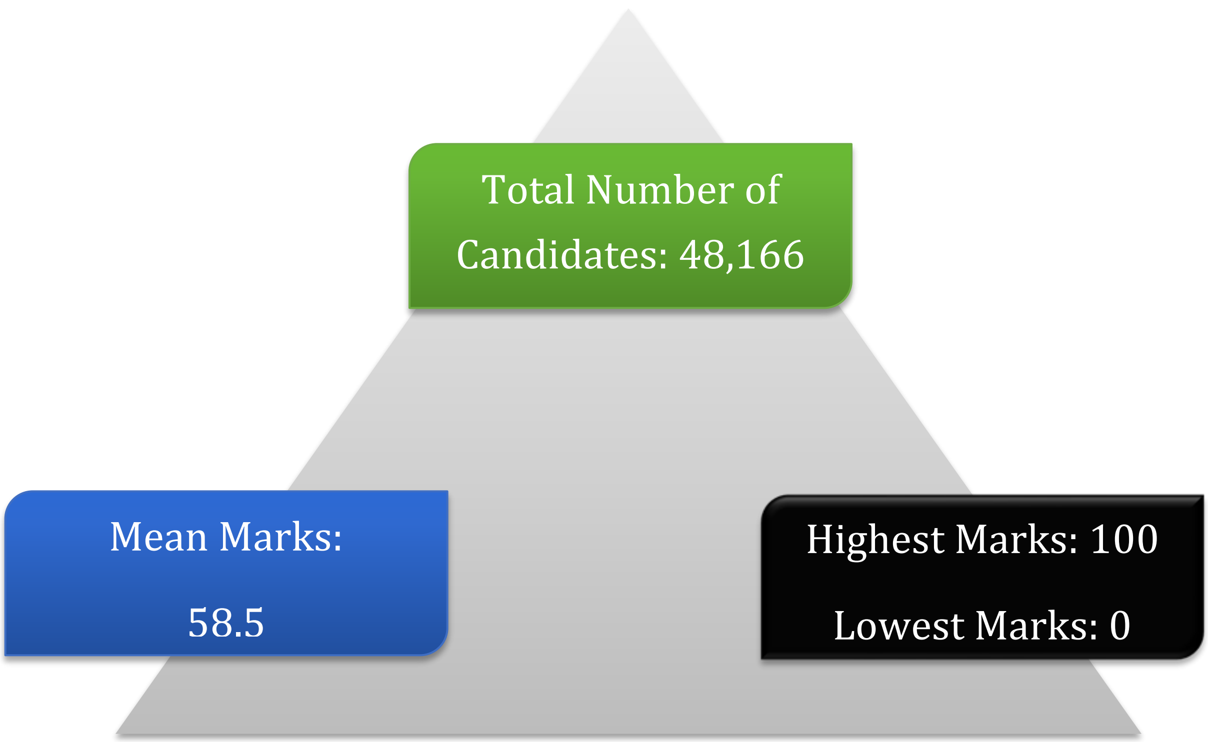
Qualitative Analysis

The purpose of the qualitative analysis is to provide insights into how candidates have performed in individual questions set in the question paper. This section is based on inputs provided by examiners from examination centres across the country. It comprises of question wise feedback on the performance of candidates in the form of *Comments of Examiners* on the common errors made by candidates along with *Suggestions for Teachers* to rectify/ reduce these errors. The *Marking Scheme* for each question has also been provided to help teachers understand the criteria used for marking. Topics in the question paper that were generally found to be difficult or confusing by candidates, have also been listed down, along with general suggestions for candidates on how to prepare for the examination/ perform better in the examination.

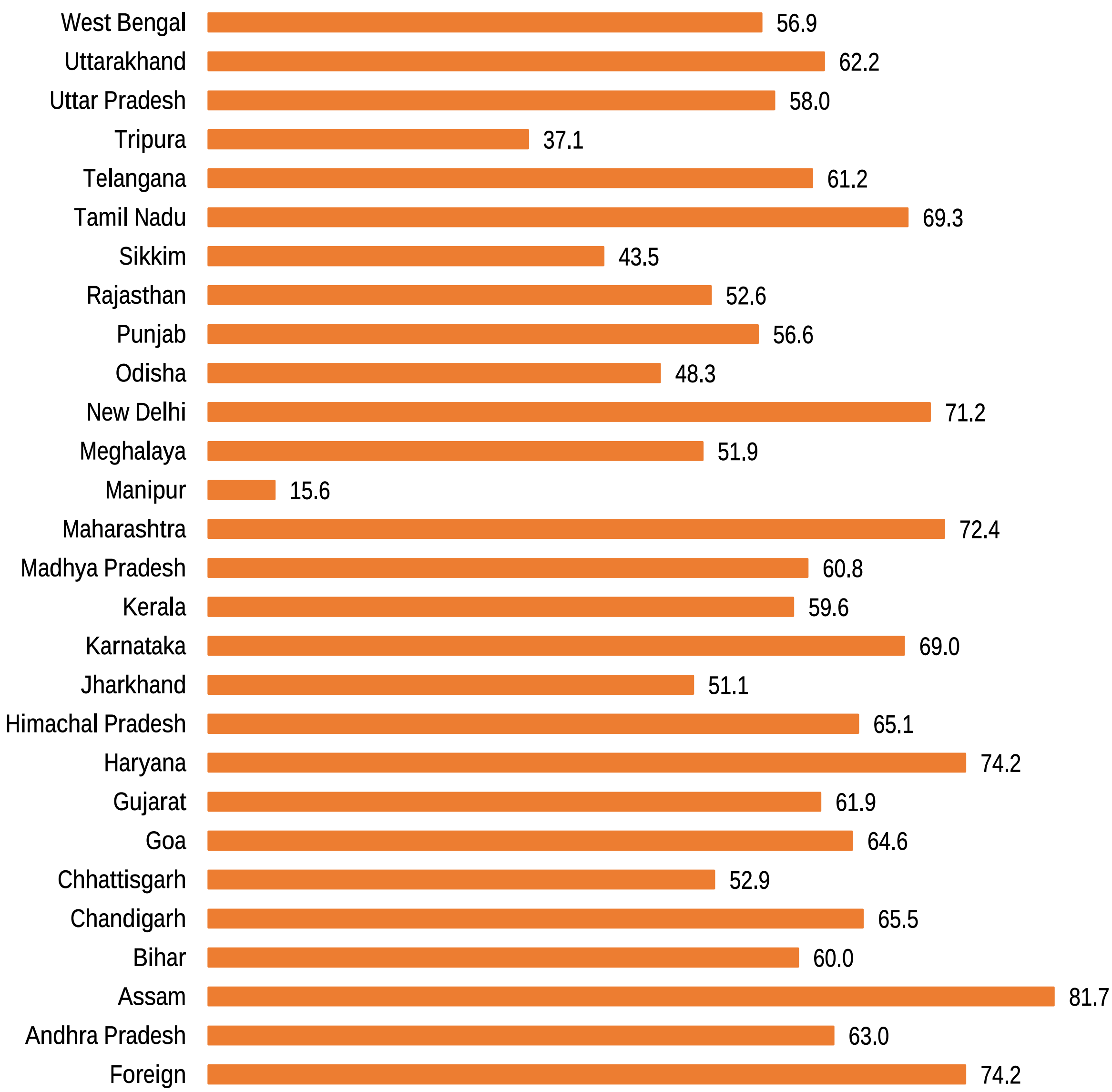
QUANTITATIVE ANALYSIS



STATISTICS AT A GLANCE



PERFORMANCE (STATE-WISE & FOREIGN)



The States of Assam, Haryana and Maharashtra secured highest mean marks. Mean marks secured by candidates studying in schools abroad were 74.2.

GENDER-WISE COMPARISON

GIRLS

Mean Marks: 61.1

Number of Candidates: 18,926

BOYS

Mean Marks: 56.9

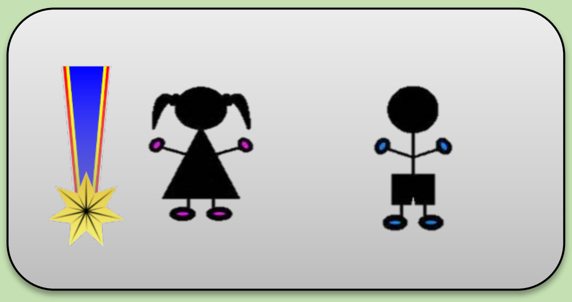
Number of Candidates: 29,240

Comparison on the basis of Gender

Gender	N	Mean	SE	t-value
Girls	18,926	61.1	0.17	18.76*
Boys	29,240	56.9	0.15	

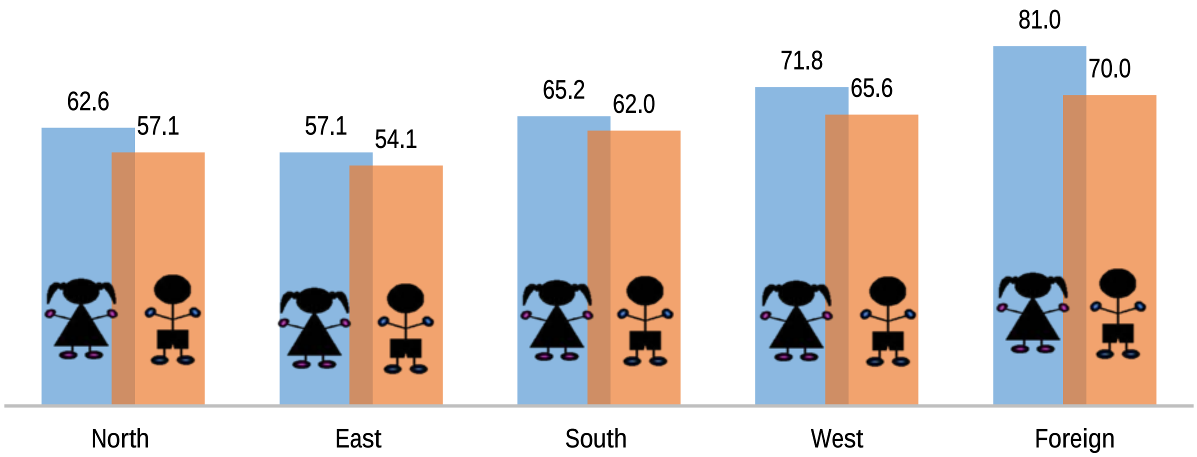
*Significant at 0.05 level

Girls performed significantly better than boys.



REGION-WISE COMPARISON

Mean Marks obtained by Boys and Girls-Region wise

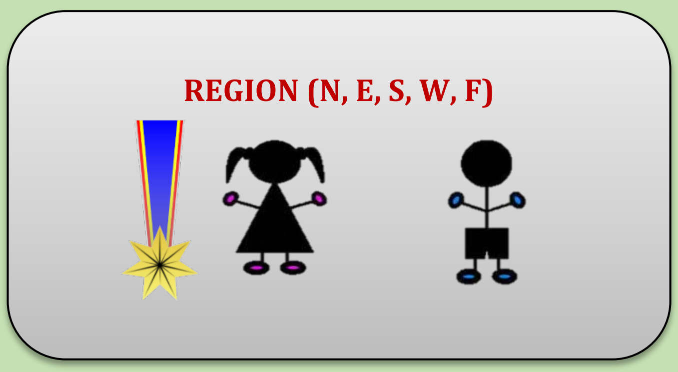


Comparison on the basis of Gender within Region

Region	Gender	N	Mean	SE	t-value
North (N)	Girls	7,215	62.6	0.26	16.11*
	Boys	13,705	57.1	0.21	
East (E)	Girls	8,338	57.1	0.26	8.40*
	Boys	11,440	54.1	0.25	
South (S)	Girls	2,326	65.2	0.41	5.25*
	Boys	2,520	62.0	0.44	
West (W)	Girls	968	71.8	0.69	6.58*
	Boys	1,445	65.6	0.64	
Foreign (F)	Girls	79	81.0	1.80	4.31*
	Boys	130	70.0	1.80	

*Significant at 0.05 level

The performance of girls was significantly better than that of boys in all the regions.



MARK RANGES : COMPARISON GENDER-WISE

Comparison on the basis of gender in top and bottom mark ranges

Marks Range	Gender	N	Mean	SE	t-value
Top Range (81-100)	Girls	4,415	89.8	0.08	-1.63
	Boys	6,245	90.0	0.07	
Bottom Range (0-20)	Girls	1,194	12.0	0.17	7.32*
	Boys	3,256	10.5	0.11	

*Significant at 0.05 level

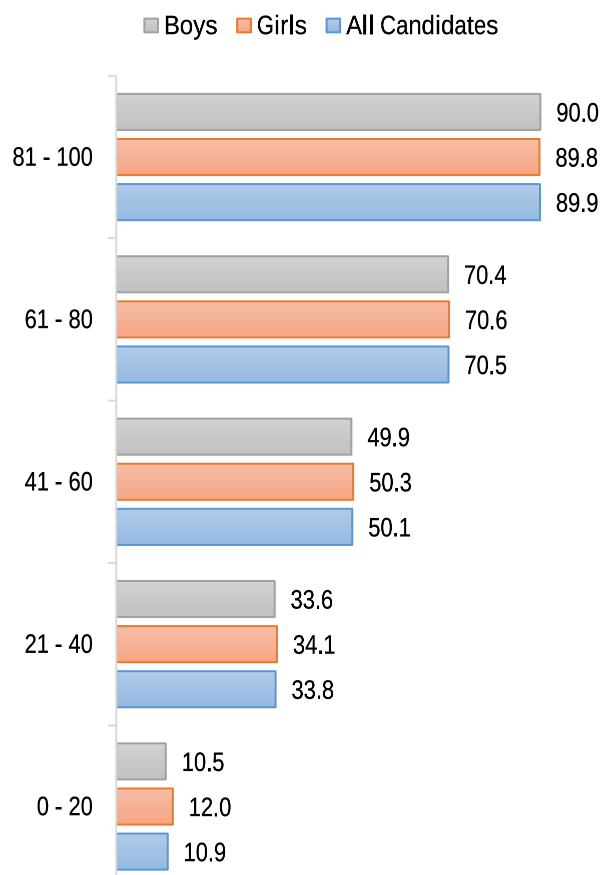
Marks Range (81-100)

No significant difference was found in the performance of girls and boys in the top marks range.

Marks Range (0-20)

Marks Range (0-20)

Performance of girls was significantly better than the performance of boys.



GRADES AWARDED : COMPARISON GENDER-WISE

Comparison on the basis of gender in Grade 1 and Grade 9

Grades	Gender	N	Mean	SE	t-value
Grade 1	Girls	2,142	94.8	0.07	-0.12
	Boys	3,141	94.8	0.05	
Grade 9	Girls	1,872	16.7	0.18	8.13*
	Boys	4,675	14.9	0.12	

*Significant at 0.05 level

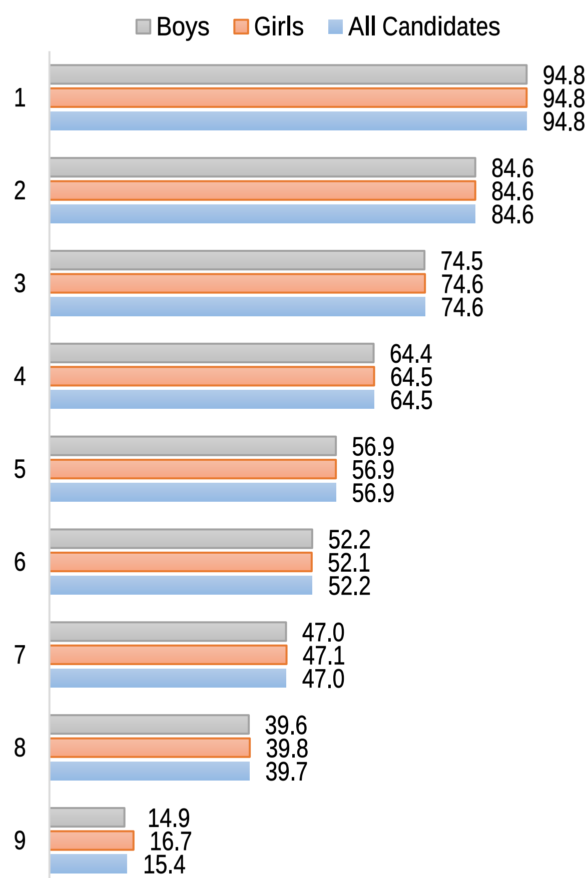
Grade 1

No significant difference was found in the performance of girls and boys.

Grade 9

Grade 9

Performance of girls was significantly better than the performance of boys .



SECTION A (80 Marks)

Question 1

[10×2]

(i) The binary operation $*$: R × R ⟶ R is defined as a $*$ b = 2a + b.
Find (2 $*$ 3) $*$ 4.

(ii) If A = $\begin{pmatrix} 5 & a \\ b & 0 \end{pmatrix}$ and A is symmetric matrix, show that $a = b$

(iii) Solve: $3tan^{-1}x + cot^{-1}x = \pi$

(iv) Without expanding at any stage, find the value of:

$$\begin{vmatrix} a & b & c \\ a+2x & b+2y & c+2z \\ x & y & z \end{vmatrix}$$

(v) Find the value of constant 'k' so that the function f(x) defined as:

$$f(x) = \begin{cases} \frac{x^2-2x-3}{x+1}, & x \neq -1 \\ k, & x = -1 \end{cases}$$

is continuous at $x = -1$.

(vi) Find the approximate change in the volume 'V' of a cube of side x metres caused by decreasing the side by 1%.

(vii) Evaluate: $\int \frac{x^3+5x^2+4x+1}{x^2} dx$.

(viii) Find the differential equation of the family of concentric circles $x^2 + y^2 = a^2$

(ix) If A and B are events such that P(A) = $\frac{1}{2}$, P(B) = $\frac{1}{3}$ and P(A∩B) = $\frac{1}{4}$, then find:

(a) P (A/B)

(b) P (B/A)

(x) In a race, the probabilities of A and B winning the race are $\frac{1}{3}$ and $\frac{1}{6}$ respectively. Find the probability of neither of them winning the race.

Comments of Examiners

(i) Many candidates used conventional multiplication operation instead of the operation as defined in the question.

(ii) A few candidates could not solve the problem based on *symmetric matrix*.

(iii) A large number of candidates attempted to solve by using formula of $3\tan^{-1}x$ but could not succeed solving further. Most candidates attempted this question using long calculations instead of using simple fundamental concepts.

(iv) Many candidates expanded the determinant to solve it, though it was clearly mentioned in the question - find the value of the determinant *without expansion at any stage*.

(v) Many candidates made errors in calculating LHL and RHL and some of the candidates could not solve it further.

(vi) Many candidates did not have an idea of approximation concept as application of differentiation and a few candidates made mistakes in writing the final answer in the correct form.

(vii) This question was attempted correctly by most of the candidates. Some of the candidates made errors in simplifying the expression before integration.

(viii) Many candidates did not have an idea of formation of differential equation and made errors while solving it.

(ix) Most of the candidates attempted this question correctly but a few got confused while presenting the solution and result in proper manner.

(x) A large number of candidates could not find the *probability of neither of them winning the race* correctly.

Suggestions for teachers

– Give adequate practice for the comprehension of well-defined mathematical operations.
– Explain basic concepts, definitions thoroughly to students and give ample practice of solving problems based on fundamental concepts.
– Clarify the conversion of inverse circular functions (one to another form) to students. Also provide sufficient practice for conversion through diagram and by using formulae.
– Train students to solve questions by using proper logic and reasoning.
– Illustrate properties of determinants in detail. Give extensive practice in different types of questions based on properties of determinants.
– Interpret the concept of continuous and discontinuous functions giving illustrations.
– Explain the concept of approximation as one of the applications of differentiation and give enough practice to students.
– Train students to simplify different types of algebraic expressions in the form of required simplest form for integration.
– Inculcate in students the fundamentals on formation of differential equations and concepts of identifying the order and degree of the equation.
– Emphasise the concept of conditional probability with illustrations.
– Discuss the concept of an event, complement of an event and the idea of finding the probability of neither of the events occurring at a time.

MARKING SCHEME

Question 1

(i)	$a*b = 2a + b$ $2*3 = 2.2 + 3 = 7$ $(2*3)*4 = 7*4$ $= 2\cdot7+4 = 18$
(ii)	$A = \begin{pmatrix} 5 & a \\ b & 0 \end{pmatrix}$ A is symmetric matrix $\rightarrow A = A^1$ $A^1 = \begin{pmatrix} 5 & b \\ a & 0 \end{pmatrix}$ $\because A = A^1 \Rightarrow \begin{pmatrix} 5 & a \\ b & 0 \end{pmatrix} = \begin{pmatrix} 5 & b \\ a & 0 \end{pmatrix}$ $\Rightarrow a = b.$
(iii)	$3\tan^{-1}x + \cot^{-1}x = \pi$ $\Rightarrow 2tan^{-1} + \tan^{-1}x + cot^{-1}x = \pi$ $2tan^{-1}x + \frac{\pi}{2} = \pi$ $2tan^{-1}x = \pi - \frac{\pi}{2} = \frac{\pi}{2}$ $tan^{-1}x = \frac{\pi}{4} \Rightarrow x = 1$
(iv)	R: $R_1 + 2R_3$ $\begin{vmatrix} a & b & c \\ a+2x & b+2y & c+2z \\ x & y & z \end{vmatrix} = \begin{vmatrix} a+2x & b+2y & c+2z \\ a+2x & b+2y & c+2z \\ x & y & z \end{vmatrix} = 0$
(v)	The function f(x) is continuous $\therefore \underset{x \to -1}{lt} \frac{x^2 - 2x - 3}{x+1} = \underset{x \to -1}{lt} \frac{(x+1)(x-3)}{x+1}$ $= \underset{x \to -1}{lt} (x-3) = -4$ $\underset{x \to -1}{lt} f(x) = f(-1)$ $\therefore k = -4$
(vi)	$v = x^3$ $\frac{dv}{dx} = 3x^2, \quad \delta x = \frac{1}{100}x$ $\delta v = \frac{dv}{dx}.\delta x = 3x^2.\frac{x}{100} = \frac{3x^3}{100} = 0 \cdot 03x^3 \quad m^3$
(vii)	$\int \frac{x^3+5x^2+4x+1}{x^2} dx$ $= \int (x + 5 + \frac{4}{x} + \frac{1}{x^2})\, dx$

	$= \frac{x^2}{2} + 5x + 4logx + \frac{-1}{x} + c$
(viii)	$x^2 + y^2 = a^2$ differentiate with respect to x $2x + 2y\frac{dy}{dx} = 0$ $\Rightarrow x + y\frac{dy}{dx} = 0$
(ix)	(a) P $(A/B) = \frac{P(A\cap \boldsymbol{B})}{P(B)} = \frac{\frac{1}{4}}{1/3} = \frac{3}{4}$
	(b) P $(B/A)\frac{P(A\cap \boldsymbol{B})}{P(A)} = \frac{\frac{1}{4}}{1/2} = \frac{2}{4} = \frac{1}{2}$
(x)	P(A) $= \frac{1}{3}$ $P(\bar{A}) = \frac{2}{3}$ P(B) $= \frac{1}{6} P(\bar{B}) = \frac{5}{6}$ Probability of neither of them winning the race: $P(\bar{A}).\,P(\bar{B})$ $= \frac{2}{3}.\frac{5}{6} = \frac{5}{9}$

Question 2

[4]

If the function f(x) = $\sqrt{2x-3}$ is invertible then find its inverse. Hence prove that $(fof^{-1})\,(x) = x$.

Comments of Examiners

A number of candidates did not read the question carefully and made an attempt to prove that the given function is invertible instead of proving $(fof^{-1})\,(x)=x$.

Suggestions for teachers

Instruct students to read the questions carefully.

MARKING SCHEME

Question 2

Let y = f(x) = $\sqrt{2x-3}$

$y^2 = 2x - 3$

$x = \frac{y^2+3}{2}$

$\Rightarrow f^{-1}(y) = \frac{y^2+3}{2}$

$f^{-1}(x) = \frac{x^2+3}{2}$

$(fof^{-1})(x) = f[f^{-1}(x)]$

$= f\left(\frac{x^2+3}{2}\right) = \sqrt{2.\frac{x^2+3}{2} - 3} = x$

Question 3

[4]

If $\tan^{-1}a + \tan^{-1}b + \tan^{-1}c = \pi$, prove that $a + b + c = abc$.

Comments of Examiners

Many candidates applied the correct formula but made errors while solving further hence, couldn't get the required result.

Suggestions for teachers

Give adequate practice in solving problems on different types of inverse trigonometric functions.

MARKING SCHEME

Question 3

$\tan^{-1}a + \tan^{-1}b + \tan^{-1}c = \pi$

$\tan^{-1}\left(\frac{a+b}{1-ab}\right) + \tan^{-1}c = \pi$

$\tan^{-1}\left[\frac{\frac{a+b}{1-ab}+c}{1-\frac{a+b}{1-ab}c}\right] = \pi$

$\tan^{-1}\frac{a+b+c-abc}{1-ab-ac-bc} = \pi$

$\Rightarrow \frac{a+b+c-abc}{1-ab-ac-bc} = \tan\pi = 0$

$\Rightarrow a+b+c = abc$

Alternate: $\tan^{-1}\left(\frac{a+b}{1-ab}\right) = \pi - \tan^{-1}c$

$\frac{a+b}{1-ab} = \tan(\pi - \tan^{-1}c)$

$\frac{a+b}{1-ab} = -c$

$a+b+c = abc$

Question 4

[4]

Use properties of determinants to solve for *x*:

$$\begin{vmatrix} x+a & b & c \\ c & x+b & a \\ a & b & x+c \end{vmatrix} = 0 \text{ and } x \neq 0.$$

Comments of Examiners

Most of the candidates made errors while applying the properties of determinants in the correct order. Hence, they failed to take common factor $(x+ a+ b+ c)$ and could not simplify the determinant.

Suggestions for teachers

Elucidate all properties of determinants and their application.

MARKING SCHEME

Question 4

$C_1 + C_2 + C_3$

$$\begin{vmatrix} x+a+b+c & b & c \\ x+a+b+c & x+b & a \\ x+a+b+c & b & x+c \end{vmatrix} = (x+a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & x+b & a \\ 1 & b & x+c \end{vmatrix}$$

$R_1 - R_3$

$$(x+a+b+c).\begin{vmatrix} 0 & 0 & -x \\ 1 & x+b & a \\ 1 & b & x+c \end{vmatrix}$$

$(x + a + b + c). [-x (b - x - b)]$

$x^2 (x + a + b + c) = 0$

$\Rightarrow x = -(a + b + c)$

Question 5

[4]

(a) Show that the function $f(x) = \begin{cases} x^2, x \le 1 \\ \frac{1}{x}, \ x > 1 \end{cases}$ is continuous at $x = 1$ but not differentiable.

OR

(b) Verify Rolle's theorem for the following function: $f(x) = e^{-x} \sin x$ on $[0, \pi]$

Comments of Examiners

(a) Many candidates successfully attempted continuity of the function correctly but failed to calculate LH derivative and RH derivative.

(b) While this question was attempted correctly by majority of the candidates, a few candidates could not write the correct intervals.

Suggestions for teachers

- Explain the concept of continuity and differentiability of a function with ample examples.
- Explain the Mean value theorem in detail including difference between the closed and open intervals with their significance by sketching the graph of the same.

MARKING SCHEME

Question 5

(a)

LHL: $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x^2 = 1$

RHL $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \frac{1}{x} = 1$

$\therefore LHL = RHL$

The f(x) is continuous at $x = 1$

$$LHD = \lim_{h \to 0^-} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0^-} \frac{(1+h)^2 - 1}{h} = 2$$

	RHD = $\underset{h \to 0^+}{lt} \frac{f(1+h)-f(1)}{h} = \underset{h \to 0^+}{lt} \frac{\frac{1}{1+h}-1}{h} = \frac{-h}{h(1+h)}$ LHD ≠ RHD ⇒f is not derivable at $x = 1$.
OR	
(b)	$f(x) = e^{-x} \sin x$ is continuous in $[0, \pi]$ $f^1(x) = e^{-x} (\cos x - \sin x)$ exists in $(0, \pi)$ $f(0) = f(\pi) = 0$ Rolle's theorem conditions are satisfied. ∴ there exists at least one value of $x = c$ such that $f^1(c) = 0$ $\therefore f^1(c) = e^{-c}(\cos c - \sin c) = 0$ $\Rightarrow cosc - sinc = 0$ $\Rightarrow c = {}^{\pi}/_{4} \epsilon (0, \pi)$

[4]

Question 6

If $x = tan\left(\frac{1}{a}log\, y\right), prove\ that\ (1 + x^2)\frac{d^2y}{dx^2} + (2x - a)\frac{dy}{dx} = 0$

Comments of Examiners

Most of the candidates successfully attempted the first derivative part but made errors in differentiating second time by applying chain rule. Some candidates could not simplify to get the required result.

Suggestions for teachers

Train students to solve different types of questions and proving the result in the process of differentiating and simplifying the equation.

MARKING SCHEME

Question 6

$x = \tan\left(\frac{1}{a}\log y\right)$

$\frac{1}{a}\log y = tan^{-1}x$

$\frac{1}{y}.\frac{dy}{dx} = a.\frac{1}{1+x^2}$

$(1+x^2)\frac{dy}{dx} = ay$

$(1+x^2)\frac{d^2y}{dx^2} + 2x.\frac{dy}{dx} = a\frac{dy}{dx}$

$(1+x^2)\frac{d^2y}{dx^2} + (2x - a)\frac{dy}{dx} = 0$

Question 7

[4]

Evaluate: $\int tan^{-1}\sqrt{x}\, dx$

Comments of Examiners

Most of the candidates made errors in substitution and simplification or applying integration by parts concept for further simplification.

Suggestions for teachers

Give adequate practice in solving problems based on integration by parts.

MARKING SCHEME

Question 7

$\int tan^{-1}\sqrt{x}dx$ let $\sqrt{x = t}$

$x = t^2$

$dx = 2tdt$

$2\int t\, tan^{-1}(t)dt$

$2\left[tan^{-1}(t).\frac{t^2}{2} - \int \frac{1}{1+t^2}.\frac{t^2}{dt}\right]$

$$= t^2.tan^{-1}(t) - \int \frac{t^2}{1+t^2}\, dt$$

$= t^2.tan^{-1}(t) - \int 1dt + \int \frac{1}{1+t^2}dt$

$= t^2.tan^{-1}(t) - t + tan^{-1}(t) + c$

$= xtan^{-1}\sqrt{x} - \sqrt{x} + tan^{-1}\sqrt{x} + c$

Question 8

[4]

(a) Find the points on the curve $y = 4x^3 - 3x + 5$ at which the equation of the tangent is parallel to the x-axis.

OR

(b) Water is dripping out from a conical funnel of semi-vertical angle $\frac{\pi}{4}$ at the uniform rate of 2 cm^2/sec in the surface, through a tiny hole at the vertex of the bottom. When the slant height of the water level is 4 cm, find the rate of decrease of the slant height of the water.

Comments of Examiners

(a) This question was solved correctly by most of the candidates. A few candidates made mistakes while finding the point of contact at which the equation of tangent to the curve where tangent is parallel to the x-axis.

(b) Many candidates considered volume as function instead of surface area. Majority of candidates did not apply proper sign though it

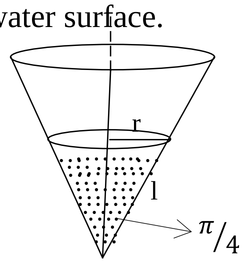
Suggestions for teachers

- Explain geometrical interpretation of differentiation with ample examples.
- Interpret all concepts of application of derivatives and allow students to practice a number of problems based on derivatives.
- Emphasise the difference between rate of increase and rate of decrease.

was specified in the question that find *the rate of decrease of the slant height of the water*.

MARKING SCHEME

Question 8

(a)	$y = 4x^3 - 3x + 5$ $\frac{dy}{dx} = 12x^3 - 3$ $\because$tangent is parallel to x – axis $\frac{dy}{dx} = 0$ $\therefore 12x^2 - 3 = 0 \Rightarrow x = \pm\frac{1}{2}$ when $x = {}^1/_2$, $y = 4\cdot\frac{1}{8} - \frac{3}{2} + 5 = \frac{1}{2} - \frac{3}{2} + 5 = \frac{1-3+10}{2} = 4$ $(\frac{1}{2}, 4)$ $x = -\frac{1}{2}$, $y = 4\cdot\frac{-1}{8} + \frac{3}{2} + 5 = \frac{-1}{2} + \frac{3}{2} + 5$ $\frac{-1+3+10}{2} = 6$ $(-\frac{1}{2}, 6)$
OR	
(b)	Let r be the radius and l be the slant height of water surface. $\sin {}^{\pi}/_4 = \frac{r}{l}$ S = $\pi r l$ $\quad and\ r = l\cdot\frac{1}{\sqrt{2}}$ $= \pi . l . \frac{l}{\sqrt{2}} = \frac{1}{\sqrt{2}}\pi l^2$ $\frac{ds}{dt} = \frac{1}{\sqrt{2}}\pi . 2l . \frac{dl}{dt}$ given that$\frac{ds}{dt} = -2\ cm^2/s$ $-2 = \frac{\pi}{\sqrt{2}} .2 l . \frac{dl}{dt}$ $at\ l = 4\ cm ,\ \frac{dl}{dt} = \frac{-\sqrt{2}}{4\pi}\ cm/s$ 

Question 9

[4]

(a) Solve: $sinx \frac{dy}{dx} - y = sinx . \tan\frac{x}{2}$

OR

(b) The population of a town grows at the rate of 10% per year. Using differential equation, find how long will it take for the population to grow 4 times.

Comments of Examiners

(a) Most candidates made errors while transforming into linear differential equation and in finding the integrating factor. Many candidates did not express the answer in terms of constant ‘C’.

(b) Not many candidates attempted this question correctly. Some candidates did not have any basic knowledge of applications of differential equations while a few candidates did not use the initial value correctly.

Suggestions for teachers

- Illustrate all types of differential equations with their applications and give adequate practice on various types of problems.
- Explain the significance of constant in the solution of differential equation.

MARKING SCHEME

Question 9

$Sin\, x \frac{dy}{dx} - y = sinx.\ tan\ \frac{x}{2}$

$\frac{dy}{dx} - y.cosec\ x = \tan\frac{x}{2}$

$$\int Pdx = -\int cosex\ dx = -\log\ tan(x/2)$$

$$\text{I.F} = e^{\int Pdx} = e^{-\log\tan\frac{x}{2}} = \frac{1}{tan^{x}/_{2}}$$

$$\text{Sol. } y.\frac{1}{tan^{x}/_{2}} = \int tan\,{}^{x}/_{2}.\frac{1}{tan^{x}/_{2}} dx + c$$

$y.\frac{1}{tan^{x}/_{2}} = x + c$

$$y.\cot{}^{x}/_{2} = x + c$$

(b) Let initial population be: P_0 and population after t years be P.

$$\frac{dp}{dt} = 10\%\ of\ P \Rightarrow \frac{dP}{P} = \frac{1}{10}\ dt$$

$$\int \frac{1}{P} dP = \frac{1}{10}\int dt$$

$$\log P = \frac{1}{10}.t + c$$

When $t = 0, P = P_0 \Rightarrow \log P_0 = c$

$$\log P = \frac{1}{10}.t + \log P_0$$

$$\log\frac{P}{P_0} = \frac{1}{10}.t$$

Since population grows 4 times, $P = 4P_0$

$\therefore\ \log 4 = \frac{1}{10}.t \Rightarrow t = 10\log 4 = 20\log 2$years

Question 10

[6]

(a) Using matrices, solve the following system of equations:

$$2x - 3y + 5z = 11$$
$$3x + 2y - 4z = -5$$
$$x + y - 2z = -3$$

OR

(b) Using elementary transformation, find the inverse of the matrix:

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

Comments of Examiners

(a) Most of the candidates attempted the question correctly. A few candidates made errors while calculating cofactors which resulted in incorrect adjoint matrix.

(b) A few candidates applied both row and column operations in the same matrix. In a number of cases, errors were committed by at the third stage of simplification and further.

Suggestions for teachers

- Explain every property of matrices with a number of examples.
- Stress upon developing logical and reasoning skills to apply the correct property of matrix.
- Ensure that basic concepts of inverse of a matrix and its properties are clear to students. Adequate practice should be given to students for solving equations by inverse matrix method.

MARKING SCHEME

Question 10

(a) Solve the following system of equations:

$$2x - 3y + 5z = 11$$
$$3x + 2y - 4z = -5$$
$$x + y - 2z = -3$$

$$\begin{pmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 11 \\ -5 \\ -3 \end{pmatrix} \quad \text{det A} = -1$$

$$AX = B$$

$$X = A^{-1}B$$

$$\text{Adj A} = \begin{pmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{pmatrix}$$

	$X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \dfrac{1}{detA}.adjA.B$ $= \frac{1}{-1}\begin{pmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{pmatrix}\begin{pmatrix} 11 \\ -5 \\ -3 \end{pmatrix}$ $= \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ $\therefore x = 1, y = 2. z = 3$
OR	
(b)	Using $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$ $R_3: R_3 - 3R_2$ $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix} A$ $R_1 \leftrightarrow R_2$ $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix} A$ $R_3: R_3 + 5R_2$ $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 5 & -3 & 1 \end{bmatrix} A$ $R_1: R_1 - 2R_2$ $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 5 & -3 & 1 \end{bmatrix} A$ $R_2: R_2 - R_3$ $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ -4 & 3 & -1 \\ 5 & -3 & 1 \end{bmatrix} A$ $R_1: R_1 + \frac{1}{2}R_3$ $\therefore \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ 5 & -3 & 1 \end{bmatrix} A$ $R_3: \frac{1}{2}R_3$

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} = \begin{bmatrix}\frac{1}{2} & -\frac{1}{2} & \frac{1}{2}\\-4 & 3 & -1\\\frac{5}{2} & -\frac{3}{2} & \frac{1}{2}\end{bmatrix} A$$

$$\therefore \quad A^{-1} = \begin{bmatrix}\frac{1}{2} & -\frac{1}{2} & \frac{1}{2}\\-4 & 3 & -1\\\frac{5}{2} & -\frac{3}{2} & \frac{1}{2}\end{bmatrix}$$

Question 11

[4]

A speaks truth in 60% of the cases, while ***B*** in 40% of the cases. In what percent of cases are they likely to contradict each other in stating the same fact?

Comments of Examiners

Most of the candidates attempted this question correctly but left the result as a fraction/decimal fraction. A few candidates made errors while applying the concept of p. q_1 +q. p_1.

Suggestions for teachers

- Advise students to read the question paper and each question carefully and note what is to be answered.
- Ensure exhaustive understanding on the concepts of probability.

MARKING SCHEME

Question 11

Let A speaks truth is event A and B speaks truth is event B.

P(A) = 60% = $\frac{60}{100}$ $\quad$ $P(\bar{A}) = \frac{40}{100}$

P(B) = 40% = $\frac{40}{100}$ $\quad$ $P(\bar{B}) = \frac{60}{100}$

Required probability = P(A). $P(\bar{B}) + P(\bar{A}).P(B)$

$$= \frac{60}{100} \times \frac{60}{100} + \frac{40}{100} \times \frac{40}{100}$$

$$= \frac{52\cancel{00}}{100\cancel{00}} = \frac{52}{100} \times 100 = 52\%$$

Question 12

[6]

A cone is inscribed in a sphere of radius 12 cm. If the volume of the cone is maximum, find its height.

Comments of Examiners

Most of the candidates attempted this question incorrectly. Some candidates could not express volume of the cone as a function in mathematical form. A number of calculation mistakes were also made while differentiating.

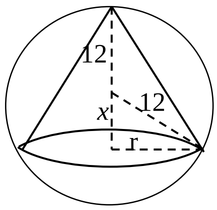
Suggestions for teachers

- Explain exhaustively the concept of maxima, minima and its application. Give practice in problems based on maxima and minima.
- Give adequate practice on mensuration related concepts and problems.

MARKING SCHEME

Question 12

Let r be the radius of cone and
Radius of sphere = 12cm and
Height of the cone = h
$\therefore\ h = 12 + x$
Volume of cone = $\frac{1}{3}\pi r^2 h$



$= \frac{1}{3}\pi(12^2 - x^2)(12 + x)$

$= \frac{1}{3}\pi(144 - x^2)(12 + x)$

$\frac{dv}{dx} = \frac{1}{3}\pi[(144 - x^2)\,1 + (12 + x)(-2x)]$

$= \frac{1}{3}\pi[144 - x^2 - 24x - 2x^2]$

$= \frac{1}{3}\pi[144 - 24x - 3x^2]$

$\frac{dv}{dx} = 0 \Rightarrow 144 - 24x - 3x^2 = 0$A1

$x^2 + 8x - 48 = 0$
$(x + 12)(x - 4) = 0$

$x = -12 \quad or\ x = 4$

at $x = 4$ $\frac{d^2v}{dx^2} = \frac{1}{3}\pi[-24 - bx] < 0$
at $x = 4$ $v.$ of cone is maximum
$\therefore$ height of cone = h = 12 + x
= 16 cm

Question 13

[6]

(a) Evaluate: $\int \frac{x-1}{\sqrt{x^2-x}}\,dx$

OR

(b) Evaluate: $\int_0^{\pi/2} \frac{cos^2 x}{1+sinx\ cosx}\,dx$

Comments of Examiners

(a) Many candidates applied incorrect techniques for solution of different types of integrals.
(b) A number of candidates could not attempt the question correctly due to incorrect use of properties of definite integrals and using incorrect methods to evaluate integrals.

Suggestions for teachers

– Teach comprehensively all standard methods of integrals, problems based on them, definite integrals, their basic properties and the problems based on them.
– Ample practice of solving different types of problems should be given to students.

MARKING SCHEME

Question 13

(a)	$= \int \frac{x-1}{\sqrt{x^2-x}} dx, \quad x-1 = \lambda(2x-1)+\mu$ $\Rightarrow \lambda = \frac{1}{2}, \ \mu = \frac{1}{2}$ $= \frac{1}{2}\int \frac{(2x-1)}{\sqrt{x^2-x}} dx - \frac{1}{2}\frac{1}{\sqrt{x^2-x}} dx$ $= \frac{1}{2}.2\sqrt{x^2-x} - \frac{1}{2}\frac{1}{\sqrt{\left(x-\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2}}$ $= \sqrt{x^2-x} - \frac{1}{2}.\log[(x-\frac{1}{2}) + \sqrt{x^2-x}]$ $= \sqrt{x^2-x} - \frac{1}{2}\log[(x-\frac{1}{2}) + \sqrt{x^2-x}] + C$
	OR
(b)	$\int_0^{\pi/2} \frac{cos^2 x}{1+sinx\cos x} dx$ I = $\int_0^{\pi/2} \frac{cos^2(\pi/2-x)}{1+\sin(\pi/x-x)\cos(\pi/2-x)} dx$ I = $\int_0^{\pi/2} \frac{sin^2 x}{1+cosx.sinx} dx$ Adding (1) and (2) 2I = $\int_0^{\pi/2} \frac{1}{1+sinx\ cosx} dx$ Dividing N & D by $\cos^2 x$ $= \int_0^{\pi/2} \frac{sec^2 xdx}{sec^2 x+tanx} dx$ Let tan x = t → $sec^2 xdx = dt$ and $x = 0$ → t = 0: $x = \pi/2$ → t = ∞ $= \int_0^{\infty} \frac{dt}{t^2+t+1}$

MARKING SCHEME

Question 14

Random variable X: 0 , 1, 2, values (defective items)

P(X=0) = $\frac{4C_4}{6C_4} = \frac{1}{15}$, $P(X = 1) = \frac{2C_1 \times 4C_3}{6C_4} = \frac{8}{15}$

P(X=2) = $\frac{2C_2 \times 4C_2}{6C_4} = \frac{6}{15}$

Probability Distribution

Defective items (x_i)	0	1	2
Probability, P_i	$\frac{1}{15}$	$\frac{8}{15}$	$\frac{6}{15}$
$P_i x_i^2$	0	$\frac{8}{15}$	$\frac{24}{15}$

Mean = $\sum p_i\, x_i = \frac{1}{15}.0 + \frac{8}{15}.1 + \frac{6}{15}.2$

$= \frac{8}{15} + \frac{12}{15} = \frac{20}{15} = \frac{4}{3}$

Variance: $\sum p_i\, x_i^2 - (\sum p_i\, x_i)^2$

$= \frac{32}{15} - \left(\frac{20}{15}\right)^2$

$= \frac{32}{15} - \frac{400}{225} = \frac{480-400}{225} = \frac{80}{225} = \frac{16}{45}$

$$= \int_0^\infty \frac{dt}{\left(t+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \left[\tan^{-1}\left(\frac{2t+1}{\sqrt{3}}\right)\right]_0^\infty$$

$2I = \frac{2\pi}{3\sqrt{3}} \Longrightarrow I = \frac{\pi}{3\sqrt{3}}$

Question 14

[6]

From a lot of 6 items containing 2 defective items, a sample of 4 items are drawn at random. Let the random variable X denote the number of defective items in the sample. If the sample is drawn without replacement, find:

(a) The probability distribution of X

(b) Mean of X

(c) Variance of X

Comments of Examiners

Most of the candidates used the concept of Binomial probability distribution to solve this problem.

Suggestions for teachers

- Ensure that students understand the concepts of probability distribution, Binomial probability distribution and their conditions.
- Sufficient practise on different types of probability distribution problems should be given.

SECTION B (20 Marks)

Question 15

[3×2]

(a) Find λ if the scalar projection of $\vec{a} = \lambda\hat{\imath} + \hat{\jmath} + 4\hat{k}$ on $\vec{b} = 2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$ is 4 units.

(b) The Cartesian equation of a line is: $2x - 3 = 3y + 1 = 5 - 6z$. Find the vector equation of a line passing through $(7, -5, 0)$ and parallel to the given line.

(c) Find the equation of the plane through the intersection of the planes
$\vec{r}.(\hat{\imath} + 3\hat{\jmath} - \hat{k}) = 9$ and $\vec{r}.(2\hat{\imath} - \hat{\jmath} + \hat{k}) = 3$ and passing through the origin.

Comments of Examiners

(a) Most of the candidates calculated dot product instead of applying projection formula.
(b) Most candidates made errors while writing the equation of a line in the symmetric form and finding direction ratios of the line. Some candidates transformed cartesian form of line into vector equation incorrectly.
(c) A few candidates wrote incorrect equation of family of plane and applied incorrect condition to find the constant.

Suggestions for teachers

- Clarify the concept of scalar projection of vector thoroughly.
- Explain the concept of transforming cartesian form to vector form and vice-versa in detail.
- Revise frequently the concepts on equation of plane through the intersection of the planes.
- Adequate practise of problems based on three-dimensional geometry must be given.

MARKING SCHEME

Question 15

(a)	Scalar projection of a on b = $\frac{a.b}{\lvert b\rvert}$ $= \frac{2\lambda+6+12}{\sqrt{4+36+9}} = \frac{2\lambda+18}{7} = 4$ $\lambda = 5$
(b)	$2x - 3 = 3y + 1 = 5 - 6z$ $\frac{x-3/2}{1/2} = \frac{y+1/3}{1/3} = \frac{z-5/6}{-1/6}$ $\Rightarrow$ Direction ratios: $\frac{1}{2}, \frac{1}{3}, -\frac{1}{6} \Rightarrow 3, 2, -1$ Equation of line passing through $(7, -5, 0)$ and parallel to the above line $\vec{r} = (7\hat{\imath} - 5\hat{\jmath} + 0\hat{k}) + \lambda\,(3\hat{\imath} + 2\hat{\jmath} - \hat{k})$
(c)	The equation of a plane through the intersection of: $\vec{r}.(\hat{\imath} + 3\hat{\jmath} - \hat{k}) = 9$ *and* $\vec{r}.(2\hat{\imath} - \hat{\jmath} + \hat{k}) = 3$ *is* $[\vec{r}.(\hat{\imath} + 3\hat{\jmath} - \hat{k}) - 9] + \lambda\,[\vec{r}.(2\hat{\imath} - \hat{\jmath} + \hat{k}) - 3] = 0$ $\vec{r}(\hat{\imath} + 3\hat{\jmath} - \hat{k}) + \lambda[\vec{r}.(2\hat{\imath} - \hat{\jmath} + \hat{k})] = 3\lambda + 9$

$\vec{r}[(\hat{\imath}+3\hat{\jmath}-\hat{k})+\lambda(2\hat{\imath}-\hat{\jmath}+\hat{k})] = 3\lambda+9$

$\because$ passing through origin.

$3\lambda+9 = 0 \Rightarrow \lambda = -3$

Required equation $\vec{r}.[-5\hat{\imath}+6\hat{\jmath}-4\hat{k}] = 0$

Alternate: $x+3y-z-9=0$ and $2x-y+z-3=0$

$$x+3y-z-9+\lambda(2x-y+z-3)=0$$

$$-9-3\lambda=0 \Rightarrow \lambda=-3$$

$-5x+6y-4z=0$

$5x-6y+4z=0$

Question 16

[4]

(a) If A, B, C are three non-collinear points with position vectors $\vec{a}, \vec{b}, \vec{c}$, respectively, then show that the length of the perpendicular from C on AB is $\frac{|(\vec{a}\times\vec{b})+(\vec{b}\times\vec{c})+(\vec{b}\times\vec{a})|}{|\vec{b}-\vec{a}|}$.

OR

(b) Show that the four points A, B, C and D with position vectors $4\hat{\imath}+5\hat{\jmath}+\hat{k}, -\hat{\jmath}-\hat{k}$, $3\hat{\imath}+9\hat{\jmath}+4\hat{k}$ and $4(-\hat{\imath}+\hat{\jmath}+\hat{k})$ respectively, are coplanar.

Comments of Examiners

(a) Very few candidates attempted this question.
(b) Majority of the candidates solved the question correctly. A few candidates made calculation errors in the process of simplification of scalar triple product.

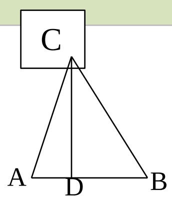
Suggestions for teachers

- Explain exhaustively about the area of triangle by using position vectors and applications based on the concept.
- Advise students to concentrate on solving a sum stepwise to avoid calculation mistakes.

MARKING SCHEME

Question 16

(a) $\Delta\, ABC =$



$= ½\,|\overrightarrow{AB}|$. (length of$\perp$from C to AB).

$= ½\,|\vec{b}-\vec{a}|$(length of $\perp$ from C to AB)

And $\Delta\, ABC = ½\,|a\times b + b\times c + c\times a)|$

$\therefore \frac{1}{2}|\vec{b}-\vec{a}|$ (length of $\perp$from C to AB) = ½ $|a\times b + b\times c + c\times a)|$

length of $\perp$ from C to AB == $\frac{|a\times b+ b\times c+c\times a)|}{|\vec{b}-\vec{a}|}$

OR

(b) $\overrightarrow{AB} = (-\hat{\jmath}-\hat{k})-(4\hat{\imath}+5\hat{\jmath}+\hat{k}) = -4\hat{\imath}-6\hat{\jmath}-2\hat{k}$

	$\overrightarrow{AC} = (3\hat{\imath} + 9\hat{\jmath} + 4\hat{k}) - (4\hat{\imath} + 5\hat{\jmath} + \hat{k}) = -\hat{\imath} + 4\hat{\jmath} + 3\hat{k}$ $\overrightarrow{AD}$=$4(-\hat{\imath} + \hat{\jmath} + \hat{k}) - (4\hat{\imath} + 5\hat{\jmath} + \hat{k}) = -8\hat{\imath} - \hat{\jmath} + 3\hat{k}$ $[\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}] = 0$ $\begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix} = -4\ (15) + 6\ (21) - 2\ (33)$ $= -60 + 126 - 66 = 0$

Question 17

[4]

(a) Draw a rough sketch of the curve and find the area of the region bounded by curve $y^2 = 8x$ and the line $x = 2$.

OR

(b) Sketch the graph of $y = |x+4|$. Using integration, find the area of the region bounded by the curve $y = |x+4|$ and $x = -6$ *and* $x = 0$.

Comments of Examiners

(a) Many candidates identified the symmetry of the curve about X-axis incorrectly and either did not apply correct limits or considered area twice the result. In most of the cases, rough sketch of the curve was not drawn correctly.

(b) Quite a few candidates attempted this question. Many candidates did not understand the concept of absolute value function and could not identify the limits of it by using the given conditions.

Suggestions for teachers

- Interpret the graphs of the standard functions in detail.
- Give sufficient practice in solving problems based on area under a given curve and straight line, etc.
- Explain well about the absolute value functions and train students to draw the rough sketch of the same by applying the given conditions.

MARKING SCHEME

Question 17

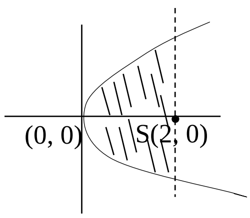
$y^2 = 4 \cdot 2 \cdot x$

$y = \sqrt{8}.\sqrt{x}$ is 1st quadrant

Area = $2\int_0^2 y dx$

= $2\sqrt{8}\int_0^2 \sqrt{x}\, dx = 4\sqrt{2}\frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^2$

$= \frac{32}{3}$ square units.



OR

$y = |x+4|$

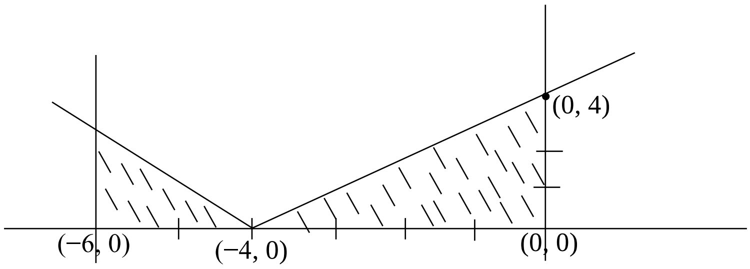
$x + 4 > 0 \Rightarrow x > -4$

$x + 4 < 0 \Rightarrow x < -4$

$y = \begin{cases} (x+4) \ \forall\, x > -4 \\ (-4-x) \ \forall\, x < -4 \end{cases}$

$\int_{-6}^{-4} -(x+4)dx + \int_{-4}^{0}(x+4)dx$

$\left[-\left(\frac{x^2}{2}+4x\right)\right]_{-6}^{-4} + \left[\frac{x^2}{2}+4x\right]_{-4}^{0} \quad = 10$ square units



Question 18

[6]

Find the image of a point having position vector: $3\hat{\imath} - 2\hat{\jmath} + \hat{k}$ in the Plane $\vec{r}.(3\hat{\imath} - \hat{\jmath} + 4\hat{k}) = 2.$

Comments of Examiners

Most of the candidates applied the condition for perpendicularity incorrectly and used incorrect direction ratios. It led to calculate incorrect foot of the perpendicular and incorrect image of the given point.

Suggestions for teachers

Ensure that the students do enough practise of problems based on applications of distance of a point etc. with reference to a line or a plane.

MARKING SCHEME

Question 18

P.V. of a point : $3i - 2j + k$

$: (3, -2, 1)$

Eq. of plane : $\vec{r}\,(3i - j + 4k) = 2$

$\Rightarrow 3x - y + 4z = 2$

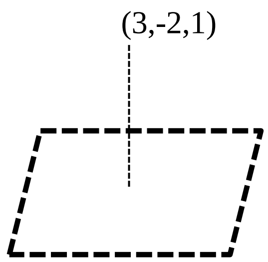
D.R. of normal to the plane:

$3, -1, 4$

$\therefore$ Eq. of straight line passing through $(3,-2,1)$ in

$\frac{x-3}{3} = \frac{y+2}{-1} = \frac{z-1}{4} = \text{say}(\lambda)$

Some value of λ the foot of the perpendicular is



$(3\lambda+3, -\lambda-2, 4\lambda+1)$

$3(3\lambda+3)-1(-\lambda-2)+4(4\lambda+1)=2$

$26\lambda+15=2$

$26\lambda=-13$

$\lambda=\frac{-13}{26}=\frac{-1}{2}$

Let (x_1, y_1, z_1) be the image of the point with reference to the given plane

$\therefore$ foot of the perpendicular $\left(\frac{x_1+3}{2}, \frac{y_1-2}{2}, \frac{z_1+1}{2}\right)=\left(\frac{3}{2}, \frac{-3}{2}, -1\right)$

$\Rightarrow x_1=0 \quad y_1=-1 \quad z_1=-3$

$(0,-1,-3)$

SECTION C (20 Marks)

Question 19

[3×2]

(a) Given the total cost function for x units of a commodity as:

$$C(x) = \frac{1}{3}x^3 + 3x^2 - 16x + 2.$$

Find:

(i) Marginal cost function

(ii) Average cost function

(b) Find the coefficient of correlation from the regression lines:

$$x - 2y + 3 = 0 \text{ and } 4x - 5y + 1 = 0.$$

(c) The average cost function associated with producing and marketing x units of an item is given by $AC = 2x - 11 + \frac{50}{x}$. Find the range of values of the output x, for which AC is increasing.

Comments of Examiners

(a) Most of the candidates attempted this part correctly except for a few who made mistakes in simplification.

(b) Many candidates made errors while applying condition to identify correct regression coefficients of the given lines - thereby, calculated incorrect correlation coefficient.

(c) Most of the candidates attempted the first part correctly but while calculating the value of x when AC was increasing, some candidates included the negative value of x.

Suggestions for teachers

- Explain the method of identifying correct regression coefficients of given lines and provide enough practice in solving problems.
- Analyse in detail about conditions of increasing and decreasing functions and simplification of problems based on these concepts.

MARKING SCHEME

Question 19

(a)	$C(x)=\frac{1}{3}x^3+3x^2-16x+2$ $M.C=\frac{d}{dx}\left[c(x)\right]=x^2+6x-16$ $A.C=\frac{c(x)}{x}=\frac{1}{3}x^2+3x-16+\frac{2}{x}$
(b)	Let $x-2y+3=0 \quad y$ on x $\Rightarrow 2y=x+3$ $y=\frac{1}{2}x+\frac{3}{2}\Rightarrow byx=\frac{1}{2}$ $4x-5y+1=0 \quad x$ on y $4x=5y+1$ $x=\frac{5}{4}y+\frac{1}{4}\Rightarrow bxy=\frac{5}{4}$ $byx\cdot bxy=\frac{1}{2}\cdot\frac{5}{4}=\frac{5}{8}<1\therefore$ Assumption of regression equation is correct $r=+\sqrt{\frac{5}{8}}=+0.79$
(c)	$AC=2x-11+\frac{50}{x}$ $\frac{d}{dx}(AC)=2-\frac{50}{x^2}$ AC increases $\frac{d}{dx}(AC)>0$ $\Rightarrow 2-\frac{50}{x^2}>0$ $x^2-25>0$ $(x+5)(x-5)>0$ $x<-5 or x>5$ $\because x$ is positive A.C. increases when $x > 5$

Question 20

[4]

(a) Find the line of regression of y on x from the following table.

x	1	2	3	4	5
y	7	6	5	4	3

Hence, estimate the value of y when $x = 6$.

OR

(b) From the given data:

Variable	x	y
Mean	6	8
Standard Deviation	4	6

and correlation coefficient: $\frac{2}{3}$. Find:

(i) Regression coefficients b_{yx} *and* b_{xy}

(ii) Regression line x on y

(iii) Most likely value of x when y = 14

Comments of Examiners

(a) Some candidates used incorrect formula to calculate regression coefficients like b_{yx} which resulted in incorrect regression equation.

(b) Many candidates applied incorrect formulae in the process of calculating b_{yx} and b_{xy}. Thereby made errors in calculating other two values also.

Suggestions for teachers

- Ensure that students learn formulae for calculating regression coefficients b_{yx} and b_{xy} correctly.
- Advise students to practice a number of problems based on regression.

MARKING SCHEME

Question 20

(a)

x	y	xy	x^2
1	7	7	1
2	6	12	4
3	5	15	9
4	4	16	16
5	3	15	25
$\sum x=15$	$\sum y=25$	$\sum xy=65$	$\sum x^2=55$

$$b_{yx}=\frac{\sum xy-\frac{\sum x.\sum y}{n}}{\sum x^2-\frac{(\sum x)^2}{n}}=\frac{65-\frac{15.25}{5}}{55-\frac{225}{5}}=-1$$

$\overline{x}=3, \overline{y}=5$

$y-\overline{y}=byx\left(x-\overline{x}\right)$

$y-5=-1\left(x-3\right)$

$y-5=x-3$

$x+y=8$ when $x=6, y=2$

OR

(b) $\overline{x}=6, \overline{y}=8,\quad \sigma_x=4, \sigma_y=6$ *and* $r=\frac{2}{3}$

	$b_{yx} = r.\frac{\sigma_y}{\sigma_x} = \frac{2}{3}.\frac{6}{4} = 1$ $b_{xy} = r.\frac{\sigma_x}{\sigma_y} = \frac{2}{3}.\frac{4}{6} = \frac{4}{9}$ Regression equation x *on* y $x - \bar{x} = bxy(y - \bar{y})$ $x - 6 = \frac{4}{9}(y - 8)$ $9x - 54 = 4y - 32$ $9x - 4y = 22$ When $y = 14$, $9x - 4 \times 14 = 22$ $9x = 78$ $x = 78/9 = 8.67$

Question 21

[4]

(a) A product can be manufactured at a total cost $C(x) = \frac{x^2}{100} + 100x + 40$, where x is the number of units produced. The price at which each unit can be sold is given by P = $\left(200 - \frac{x}{400}\right)$. Determine the production level x at which the profit is maximum. What is the price per unit and total profit at the level of production?

OR

(b) A manufacturer's marginal cost function is $\frac{500}{\sqrt{2x+25}}$. Find the cost involved to increase production from 100 units to 300 units.

Comments of Examiners

(a) Many candidates considered Price function as Revenue function which led to entire solution being incorrect.

(b) A few candidates made mistakes in integration part and substituted incorrect limits.

Suggestions for teachers

- Explain thoroughly with illustrations the basic difference among Price function, Cost function and Revenue function, etc.
- Interpret all Marginal functions and concepts of application of calculus in Commerce and Economics with examples.

MARKING SCHEME

Question 21

(a)

$$R(x) = P \cdot x = \left(200 - \frac{x}{400}\right) \cdot x = 200x - \frac{x^2}{400}$$

$$P(1) = R(x) - C(x)$$

$$= \left(200x - \frac{x^2}{400}\right) - \left(\frac{x^2}{100} + 100x + 40\right)$$

$$= 100x - \frac{x^2}{80} - 40$$

$$\frac{d}{dx}\left[P(x)\right] = 100 - \frac{2x}{80} = 100 - \frac{x}{40}$$

$$\frac{d^2}{dx^2}\left[P(x)\right] = 0$$

$$\Rightarrow x = 4000$$

$$\frac{d^2}{dx^2}\left[P(x)\right] = \frac{-1}{40} < 0$$

∴ P(x) is maximum at x = 4000

$$x = 4000 \quad P = \left(200 - \frac{x}{400}\right) = \left(200 - \frac{4000}{400}\right) = Rs.190$$

Maximum profit = $100x - \frac{x^2}{80} - 40$

$$= 100.4000 - \frac{(4000)^2}{80} - 40 = Rs.1,99,960$$

OR

(b)

Given: $MC = \frac{500}{\sqrt{2x+25}}$

Total increased cost when x increases from 100 to 300 units

$$c(200) - c(100) = \int_{100}^{300} MC(x)dx = \int_{100}^{300} \frac{500}{\sqrt{2x+25}}dx$$

$$500\left[\sqrt{2x+25}\right]_{100}^{300} = 500\left[\sqrt{625} - \sqrt{225}\right] = 500(25-15)$$

$$= 5000$$

Required cost increase = Rs.5000

Question 22 **[6]**

A manufacturing company makes two types of teaching aids ***A*** and ***B*** of Mathematics for Class *X*. Each type of ***A*** requires 9 labour hours for fabricating and 1 labour hour for finishing. Each type of ***B*** requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available per week are 180 and 30 respectively. The company makes a profit of ₹ 80 on each piece of type ***A*** and ₹ 120 on each piece of type ***B***. How many pieces of type ***A*** and type ***B*** should be manufactured per week to get a maximum profit? Formulate this as Linear Programming Problem and solve it. Identify the feasible region from the rough sketch.

Comments of Examiners

A number of candidates could not form the correct inequalities subject to given constraints. Some noted incorrect corner points to find optimum value. A few candidates did not draw rough sketch of the feasible region.

Suggestions for teachers

– Explain how to frame inequalities using given constraints and finding corner points by solving inequalities.
– Adequate practice in different types of problems based on inequality must be provided on a regular basis.

MARKING SCHEME

Question 22

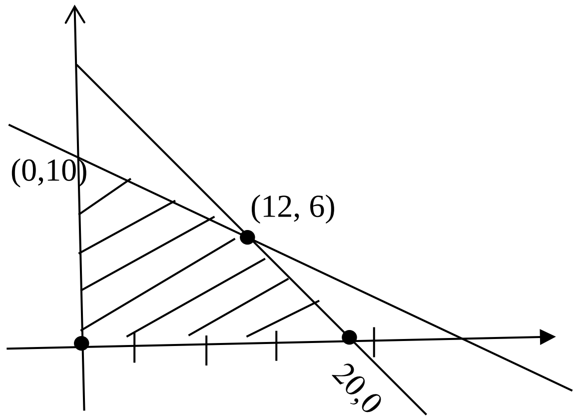
Let x and y be the number of teaching aids of type A and type B respectively

$z = 80x + 120y$

$9x + 12y \leq 180$

$x + 3y \leq 30$

$x \geq 0$ and $y \geq 0$



Solving Constants: $9x + 12y \leq 180$, $x + 3y \leq 30$

$(0,0), (20,0), (0,10)$ and $(12,6)$

At $(0,0)$ $\quad z = 80.0 + 120.0 = 0$

$(20,0)$ $\quad z = 80.20 + 120.0 = 1600$

$(0,10)$ $\quad z = 80.0 + 120.10 = 1200$

$(12,6)$ $\quad z = 80.12 + 120.6 = 1680$

Hence z is maximum at (12,6)

$\therefore x = 12$ and $y = 6$

Maximum z = Rs.1680.

Note: For questions having more than one correct answer/solution, alternate correct answers/solutions, apart from those given in the marking scheme, have also been accepted.

GENERAL COMMENTS

Topics found difficult by candidates

- Application of derivatives including Maxima and Minima.
- Integrals, and curve sketching.
- Vectors, interchange of vector equation to Cartesian equation and vice-versa.
- Probability and Probability distribution.
- Inverse circular functions.

Concepts in which candidates got confused

- Open and closed interval of mean value theorems.
- Product and sum rule of probability and dependent and independent events.
- Dot and Cross product of vectors, Projection of a vector.
- Properties of definite integrals.

Suggestions for candidates

- Avoid selective study. Study the entire syllabus thoroughly and revise from time to time.
- Revise the concepts of Class XI and integrate them with the Class XII syllabus.
- Clarify the concepts of each chapter/topic with the help of your teacher.
- Learn the Formulae related to every topic after acquiring thorough understanding of each symbol used.
- Revise all topics and formulae involved and make a chapter wise or topic-wise list of these.
- Time management is important while attempting the paper. Practise solving papers within a stipulated time.
- Practise mock papers by following the time management with the guidance of the teacher.